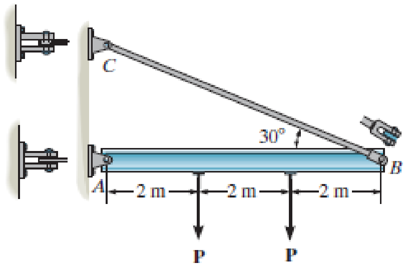
MECE202 STRENGTH OF MATERIALS (SELECTED PROBLEMS)

CHAPTER 1

•1–61. Determine the maximum magnitude P of the load the beam will support if the average shear stress in each pin is not to allowed to exceed 60 MPa. All pins are subjected to double shear as shown, and each has a diameter of 18 mm.



Referring to the FBD of member AB, Fig. a,

$\zeta + \Sigma M_A = 0; \quad F_{BC} \sin 30°(6) - P(2) - P(4) = 0 \quad F_{BC} = 2P$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 2P \cos 30° = 0 \quad A_x = 1.732P$

$+\uparrow \Sigma F_y = 0; \quad A_y - P - P + 2P \sin 30° = 0 \quad A_y = P$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.732P)^2 + P^2} = 2P$$

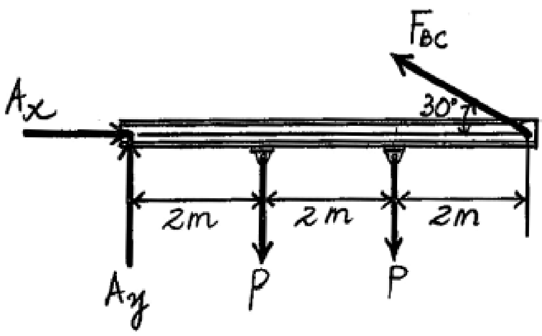
All pins are subjected to same force and double shear. Referring to the FBD of the pin, Fig. b,

$$V = \frac{F}{2} = \frac{2P}{2} = P$$

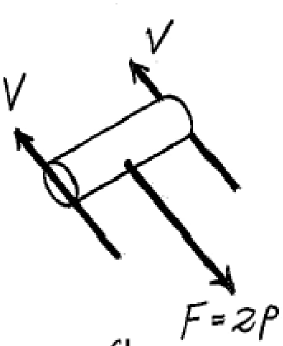
The cross-sectional area of the pin is $A = \frac{\pi}{4}(0.018^2) = 81.0(10^{-6})\pi\ \text{m}^2$. Thus,

$$\tau_{allow} = \frac{V}{A}; \quad 60(10^6) = \frac{P}{81.0(10^{-6})\pi}$$

$$P = 15\,268\ \text{N} = 15.3\ \text{kN} \qquad \textbf{Ans.}$$

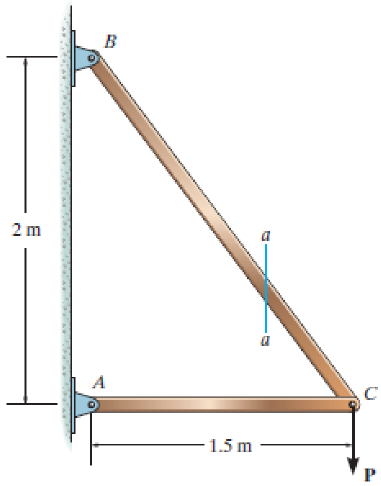


(a)



(b)

1–66. Determine the largest load **P** that can be a applied to the frame without causing either the average normal stress or the average shear stress at section *a–a* to exceed $\sigma = 150$ MPa and $\tau = 60$ MPa, respectively. Member *CB* has a square cross section of 25 mm on each side.



Analyse the equilibrium of joint *C* using the FBD Shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\frac{4}{5}\right) - P = 0 \quad F_{BC} = 1.25P$$

Referring to the FBD of the cut segment of member *BC* Fig. *b*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_{a-a} - 1.25P\left(\frac{3}{5}\right) = 0 \quad N_{a-a} = 0.75P$$

$$+\uparrow \Sigma F_y = 0; \quad 1.25P\left(\frac{4}{5}\right) - V_{a-a} = 0 \quad V_{a-a} = P$$

The cross-sectional area of section *a–a* is $A_{a-a} = (0.025)\left(\frac{0.025}{3/5}\right) = 1.0417(10^{-3})\text{m}^2$. For Normal stress,

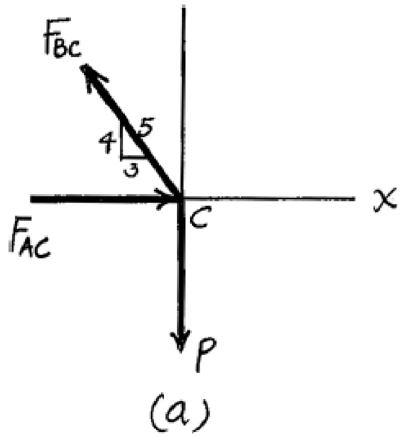
$$\sigma_{\text{allow}} = \frac{N_{a-a}}{A_{a-a}}; \quad 150(10^6) = \frac{0.75P}{1.0417(10^{-3})}$$

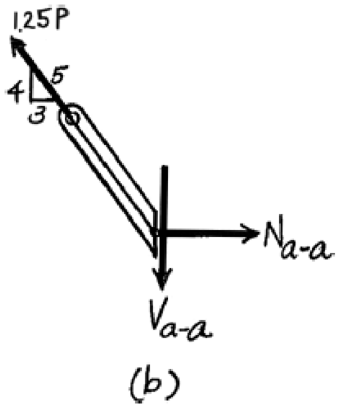
$$P = 208.33(10^3)\text{ N} = 208.33\text{ kN}$$

For Shear Stress

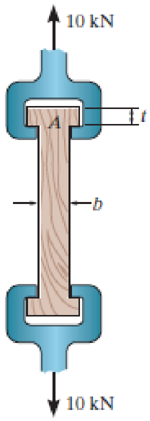
$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 60(10^6) = \frac{P}{1.0417(10^{-3})}$$

$$P = 62.5(10^3)\text{ N} = 62.5\text{ kN (Controls!)}$$ **Ans.**





•**1–77.** The wood specimen is subjected to the pull of 10 kN in a tension testing machine. If the allowable normal stress for the wood is $(\sigma_t)_{\text{allow}} = 12$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 1.2$ MPa, determine the required dimensions b and t so that the specimen reaches these stresses simultaneously. The specimen has a width of 25 mm.



Allowable Shear Stress: Shear limitation

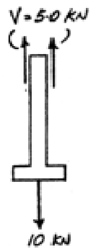
$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 1.2\left(10^6\right) = \frac{5.00(10^3)}{(0.025)\,t}$$

$$t = 0.1667 \text{ m} = 167 \text{ mm} \qquad \textbf{Ans.}$$

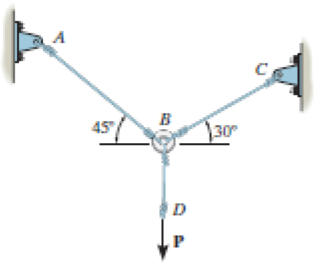
Allowable Normal Stress: Tension limitation

$$\sigma_{\text{allow}} = \frac{P}{A}; \qquad 12.0\left(10^6\right) = \frac{10(10^3)}{(0.025)\,b}$$

$$b = 0.03333 \text{ m} = 33.3 \text{ mm} \qquad \textbf{Ans.}$$



1–82. The three steel wires are used to support the load. If the wires have an allowable tensile stress of $\sigma_{allow} = 165$ MPa, determine the required diameter of each wire if the applied load is $P = 6$ kN.



The force in wire BD is equal to the applied load; ie, $F_{BD} = P = 6$ kN. Analysing the equilibrium of joint B by referring to its FBD, Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} \cos 30° - F_{AB} \cos 45° = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 30° + F_{AB} \sin 45° - 6 = 0 \qquad$ **(2)**

Solving Eqs. (1) and (2),

$F_{AB} = 5.379$ kN $\qquad F_{BC} = 4.392$ kN

For wire BD,

$$\sigma_{allow} = \frac{F_{BD}}{A_{BD}}; \qquad 165(10^6) = \frac{6(10^3)}{\frac{\pi}{4} d_{BD}^2}$$

$d_{BD} = 0.006804 \text{ m} = 6.804$ mm

Use $d_{BD} = 7.00$ mm **Ans.**

For wire AB,

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 165(10^6) = \frac{5.379(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

$d_{AB} = 0.006443 \text{ m} = 6.443$ mm

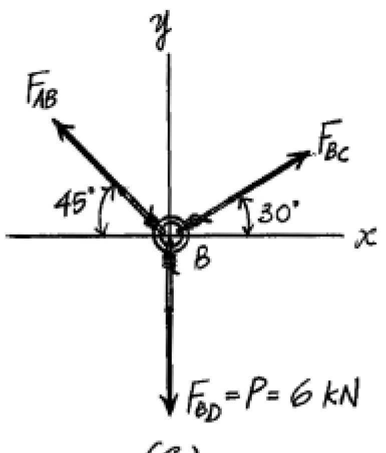
Use $d_{AB} = 6.50$ mm **Ans.**

For wire BC,

$$\sigma_{allow} = \frac{F_{BC}}{A_{BC}}; \qquad 165(10^6) = \frac{4.392(10^3)}{\frac{\pi}{4} d_{BC}^2}$$

$d_{BC} = 0.005822 \text{ m} = 5.822$ mm

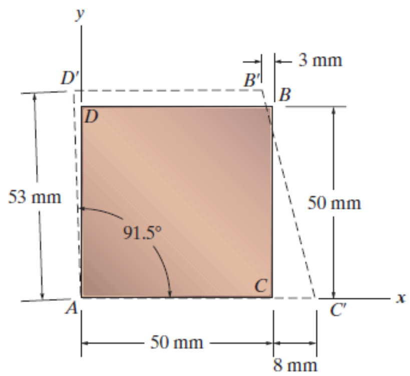
$d_{BC} = 6.00$ mm **Ans.**



(a)

CHAPTER 2

***2–16.** The square deforms into the position shown by the dashed lines. Determine the average normal strain along each diagonal, AB and CD. Side $D'B'$ remains horizontal.



Geometry:

$$AB = CD = \sqrt{50^2 + 50^2} = 70.7107 \text{ mm}$$

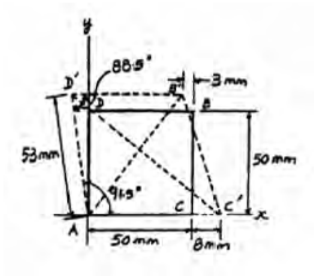
$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58)\cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

$$B'D' = 50 + 53\sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$AB' = \sqrt{53^2 + 48.3874^2 - 2(53)(48.3874)\cos 88.5°}$$

$$= 70.8243 \text{ mm}$$



Average Normal Strain:

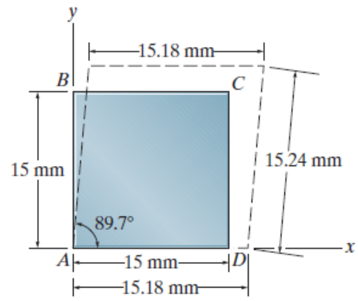
$$\varepsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{70.8243 - 70.7107}{70.7107} = 1.61\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

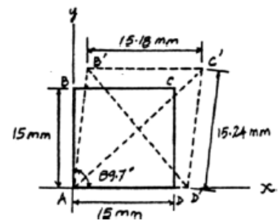
$$\varepsilon_{CD} = \frac{C'D' - CD}{CD}$$

$$= \frac{79.5860 - 70.7107}{70.7107} = 126\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

2–23. A square piece of material is deformed into the dashed parallelogram. Determine the average normal strain that occurs along the diagonals AC and BD.



Geometry:



$$AC = BD = \sqrt{15^2 + 15^2} = 21.2132 \text{ mm}$$

$$AC' = \sqrt{15.18^2 + 15.24^2 - 2(15.18)(15.24)\cos 90.3°}$$

$$= 21.5665 \text{ mm}$$

$$B'D' = \sqrt{15.18^2 + 15.24^2 - 2(15.18)(15.24)\cos 89.7°}$$

$$= 21.4538 \text{ mm}$$

Average Normal Strain:

$$\varepsilon_{AC} = \frac{AC' - AC}{AC} = \frac{21.5665 - 21.2132}{21.2132}$$

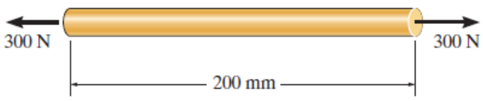
$$= 0.01665 \text{ mm/mm} = 16.7\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

$$\varepsilon_{BD} = \frac{B'D' - BD}{BD} = \frac{21.4538 - 21.2132}{21.2132}$$

$$= 0.01134 \text{ mm/mm} = 11.3\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

CHAPTER 3

•**3–25.** The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.697 \text{ MPa}$$

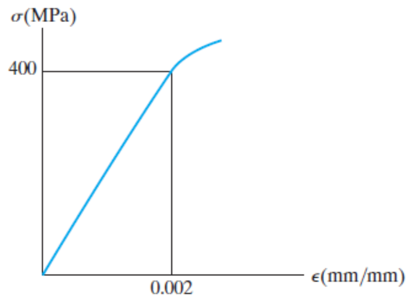
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{1.697(10^6)}{2.70(10^9)} = 0.0006288$$

$$\delta = \varepsilon_{long} L = 0.0006288\,(200) = 0.126 \text{ mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{lat} = -V \varepsilon_{long} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \varepsilon_{lat} d = -0.0002515\,(15) = -0.00377 \text{ mm} \qquad \textbf{Ans.}$$

3–27. The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. When the applied load on the specimen is 50 kN, the diameter is 12.99265 mm. Determine Poisson's ratio for the material.



Normal Stress:

$$\sigma = \frac{P}{A} = \frac{50(10^3)}{\frac{\pi}{4}(0.013^2)} = 376.70 \text{ Mpa}$$

Normal Strain: From the stress–strain diagram, the modulus of elasticity $E = \frac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's law

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{376.70(10^6)}{200(10^4)} = 1.8835\left(10^{-3}\right) \text{ mm/mm}$$

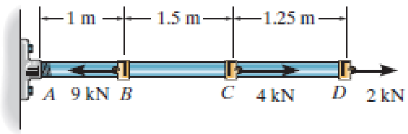
$$\varepsilon_{lat} = \frac{d - d_0}{d_0} = \frac{12.99265 - 13}{13} = -0.56538\left(10^{-3}\right) \text{ mm/mm}$$

Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio.

$$V = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{-0.56538(10^{-3})}{1.8835(10^{-3})} = 0.300 \qquad \textbf{Ans.}$$

CHAPTER 4

4–3. The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 50 mm², determine the displacement of its end D. Neglect the size of the couplings at B, C, and D.



The normal forces developed in segments AB, BC and CD are shown in the $FBDS$ of each segment in Fig. a, b and c, respectively.

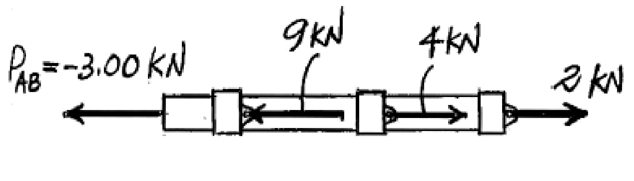
The cross-sectional areas of all the segments are $A = \left(50\ \text{mm}^2\right)\left(\dfrac{1\ \text{m}}{1000\ \text{mm}}\right)^2 = 50.0(10^{-6})\ \text{m}^2$.

$$\delta_D = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC} + P_{CD}\,L_{CD}\right)$$

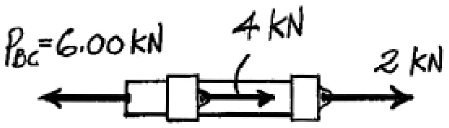
$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5) + 2.00(10^3)(1.25)\right]$$

$$= 0.850(10^{-3})\ \text{m} = 0.850\ \text{mm}$$ **Ans.**

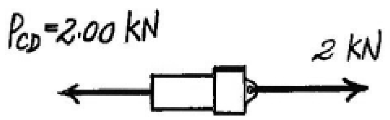
The positive sign indicates that end D moves away from the fixed support.



(a)

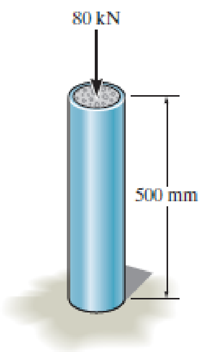


(b)



(c)

•**4–33.** The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the average normal stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. $E_{st} = 200$ GPa, $E_c = 24$ GPa.

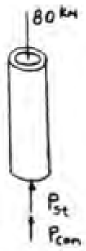


$+\uparrow \Sigma F_y = 0;\qquad P_{st} + P_{con} - 80 = 0$ **(1)**

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.08^2 - 0.07^2)\,(200)\,(10^9)} = \frac{P_{con}\,L}{\frac{\pi}{4}(0.07^2)\,(24)\,(10^9)}$$

$P_{st} = 2.5510\,P_{con}$ **(2)**



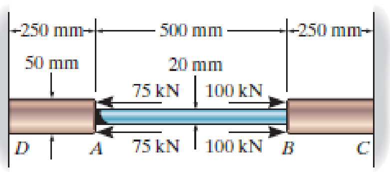
Solving Eqs. (1) and (2) yields

$$P_{st} = 57.47\text{ kN}\qquad P_{con} = 22.53\text{ kN}$$

$\sigma_{st} = \dfrac{P_{st}}{A_{st}} = \dfrac{57.47\,(10^3)}{\frac{\pi}{4}\,(0.08^2 - 0.07^2)} = 48.8\text{ MPa}$ **Ans.**

$\sigma_{con} = \dfrac{P_{con}}{A_{con}} = \dfrac{22.53\,(10^3)}{\frac{\pi}{4}\,(0.07^2)} = 5.85\text{ MPa}$ **Ans.**

***4–36.** The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the average normal stress in each segment due to the applied load.



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_C - F_D + 75 + 75 - 100 - 100 = 0$

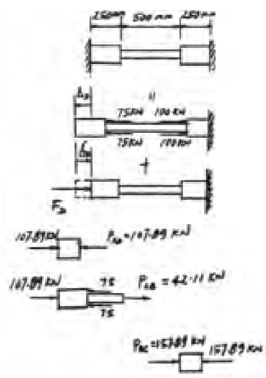
$F_C - F_D - 50 = 0$ **(1)**

$$\overset{+}{\rightarrow}\qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(0.25)}{\frac{\pi}{4}(0.05^2)(101)(10^9)}$$

$$- \frac{F_D(0.5)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.02^2)(200)(10^9)}$$

$$F_D = 107.89\text{ kN}$$



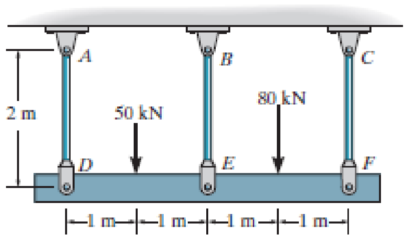
From Eq. (1), $F_C = 157.89$ kN

$\sigma_{AD} = \dfrac{P_{AD}}{A_{AD}} = \dfrac{107.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 55.0\text{ MPa}$ **Ans.**

$\sigma_{AB} = \dfrac{P_{AB}}{A_{AB}} = \dfrac{42.11(10^3)}{\frac{\pi}{4}(0.02^2)} = 134\text{ MPa}$ **Ans.**

$\sigma_{BC} = \dfrac{P_{BC}}{A_{BC}} = \dfrac{157.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 80.4\text{ MPa}$ **Ans.**

4–55. The three suspender bars are made of A-36 steel and have equal cross-sectional areas of 450 mm². Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



Referring to the *FBD* of the rigid beam, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \quad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0 \qquad$ **(1)**

$\circlearrowleft + \Sigma M_D = 0; \quad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0 \qquad$ **(2)**

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\not{L}}{\not{A}\not{E}} = \frac{1}{2}\left(\frac{F_{AD}\not{L}}{\not{A}\not{E}} + \frac{F_{CF}\not{L}}{\not{A}\not{E}}\right)$$

$$F_{AD} + F_{CF} = 2\,F_{BE} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

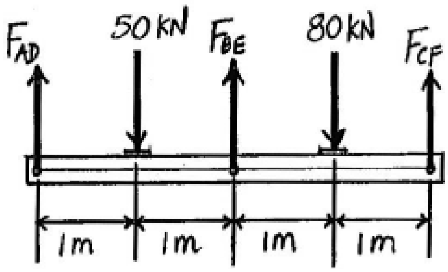
$F_{BE} = 43.33(10^3)$ N $\qquad F_{AD} = 35.83(10^3)$ N $\qquad F_{CF} = 50.83(10^3)$ N

Thus,

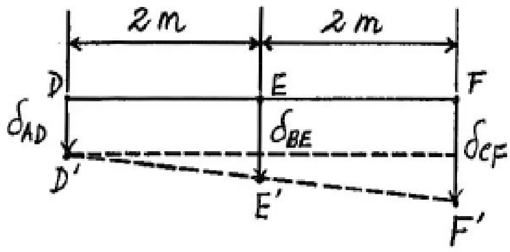
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CF} = 113 \text{ MPa} \qquad \textbf{Ans.}$$

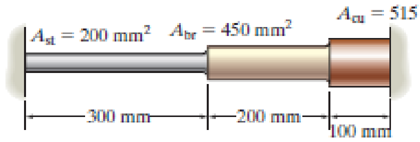


(a)



(b)

•4–69. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°C$. The material properties and cross-sectional area of each bar are given in the figure.

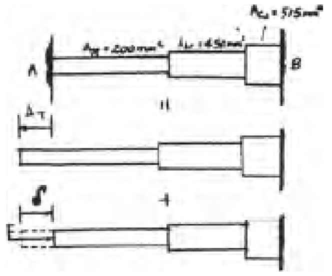


$(\overset{+}{\leftarrow}) \qquad 0 = \Delta_T - \delta$

$$0 = 12(10^{-6})(6)(0.3) + 21(10^{-6})(6)(0.2) + 17(10^{-6})(6)(0.1)$$

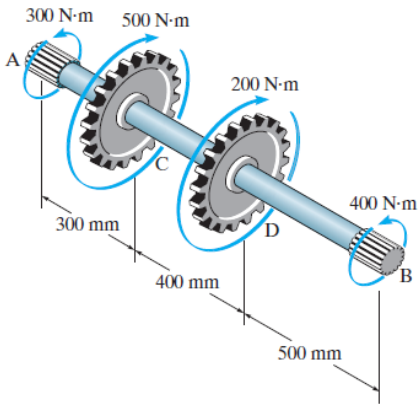
$$-\frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.2)}{450(10^{-6})(100)(10^9)} - \frac{F(0.1)}{515(10^{-6})(120)(10^9)}$$

$F = 4203 \text{ N} = 4.20 \text{ kN}$ **Ans.**



CHAPTER 5

*5–8. The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.

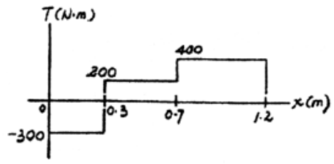


Internal Torque: As shown on torque diagram.

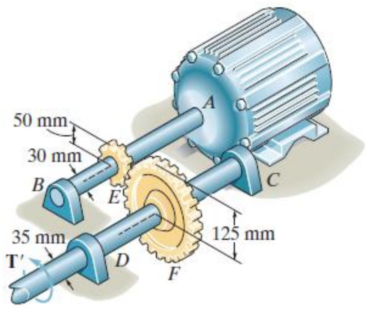
Maximum Shear Stress: From the torque diagram $T_{max} = 400\ \text{N}\cdot\text{m}$. Then, applying torsion Formula.

$$\tau^{abs}_{max} = \frac{T_{max}\,c}{J}$$

$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5\ \text{MPa} \qquad \textbf{Ans.}$$



*5–12. The motor delivers a torque of 50 N · m to the shaft *AB*. This torque is transmitted to shaft *CD* using the gears at *E* and *F*. Determine the equilibrium torque **T**′ on shaft *CD* and the maximum shear stress in each shaft. The bearings *B*, *C*, and *D* allow free rotation of the shafts.



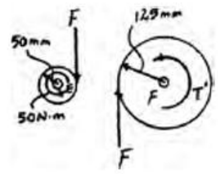
Equilibrium:

$\circlearrowleft + \Sigma M_E = 0; \qquad 50 - F(0.05) = 0 \qquad F = 1000\ \text{N}$

$\circlearrowleft + \Sigma M_F = 0; \qquad T' - 1000(0.125) = 0$

$$T' = 125\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

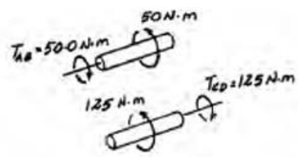
Internal Torque: As shown on FBD.



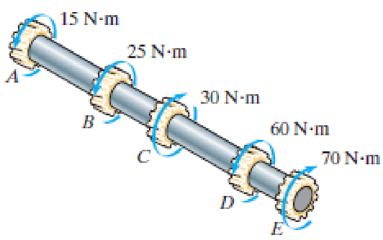
Maximum Shear Stress: Applying torsion Formula.

$$(\tau_{AB})_{max} = \frac{T_{AB}\,c}{J} = \frac{50.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 9.43\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{max} = \frac{T_{CD}c}{J} = \frac{125(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 14.8\ \text{MPa} \qquad \textbf{Ans.}$$



5–15. The solid shaft is made of material that has an allowable shear stress of $\tau_{allow} = 10$ MPa. Determine the required diameter of the shaft to the nearest mm.

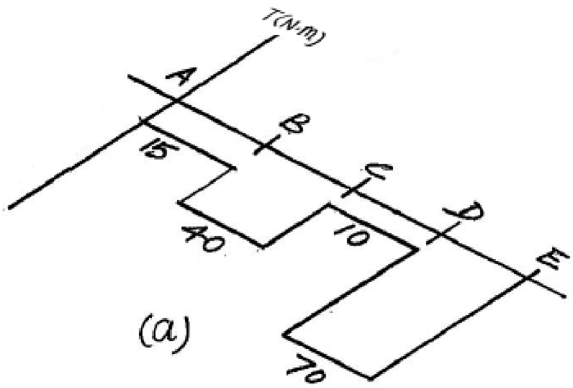


The internal torques developed in each segment of the shaft are shown in the torque diagram, Fig. a.

Segment DE is critical since it is subjected to the greatest internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$. Thus,

$$\tau_{allow} = \frac{T_{DE}\,c}{J}; \qquad 10(10^6) = \frac{70\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4}$$

$$d = 0.03291 \text{ m} = 32.91 \text{ mm} = 33 \text{ mm} \qquad \textbf{Ans.}$$



(a)