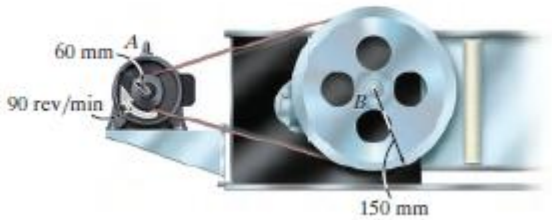
MECE202 STRENGTH OF MATERIALS (SELECTED PROBLEMS)

CHAPTER 5

5–38. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{allow} = 85$ MPa.



Internal Torque: For shafts A and B

$$\omega_A = 90\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 3.00\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\ \text{N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\ \text{N}\cdot\text{m}$$

Allowable Shear Stress: For shaft A

$$\tau_{max} = \tau_{allow} = \frac{T_A\,c}{J}$$

$$85\left(10^6\right) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\ \text{m} = 12.4\ \text{mm} \qquad \textbf{Ans.}$$

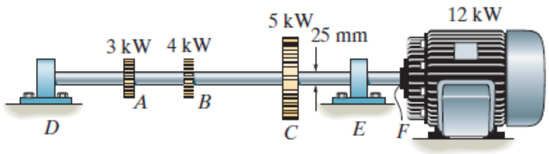
For shaft B

$$\tau_{max} = \tau_{allow} = \frac{T_B\,c}{J}$$

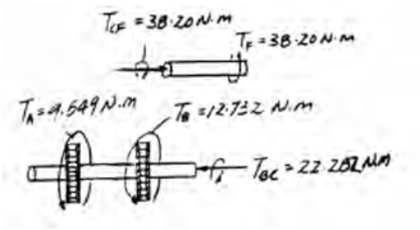
$$85\left(10^6\right) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\ \text{m} = 16.8\ \text{mm} \qquad \textbf{Ans.}$$

5–39. The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.



$$\omega = 50\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\ \pi\ \text{rad/s}$$



$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\ \pi} = 38.20\ \text{N}\cdot\text{m}$$

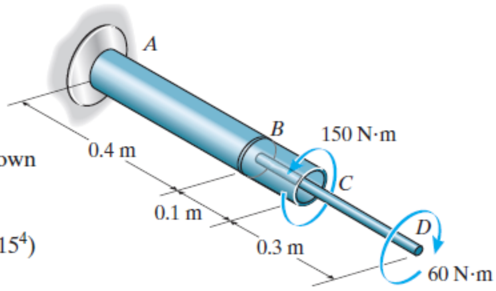
$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\ \pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\ \pi} = 12.73\ \text{N}\cdot\text{m}$$

$$(\tau_{\max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

$$(\tau_{\max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\ \text{MPa}$$ **Ans.**

5–55. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at B. Determine the angle of twist at C. The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



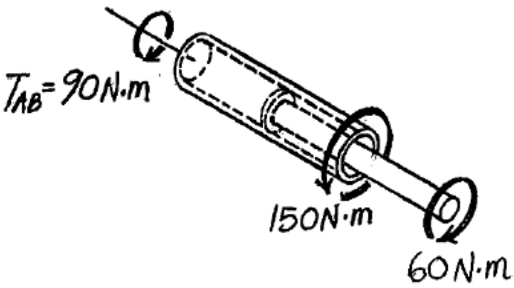
The internal torques developed in segments AB and BC of the assembly are shown in Figs. a and b.

The polar moment of inertia of the tube is $J = \frac{\pi}{2}(0.02^4 - 0.015^4) = 54.6875\,(10^{-9})\pi\ \text{m}^4$. Thus,

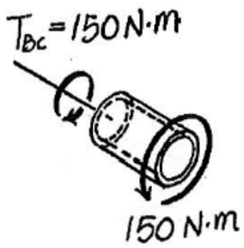
$$\phi_C = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{J G_{st}} + \frac{T_{BC} L_{BC}}{J\, G_{st}}$$

$$= \frac{1}{54.6875(10^{-9})\pi\,[75(10^9)]}\left[90(0.4) + 150(0.1)\right]$$

$$= 0.003958\ \text{rad} = 0.227^\circ$$ **Ans.**

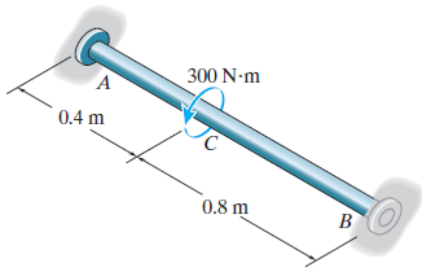


(a)



(b)

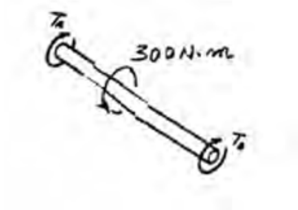
•5–77. The A-36 steel shaft has a diameter of 50 mm and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB of the shaft.



Equilibrium:

$$T_A + T_B - 300 = 0 \qquad [1]$$

Compatibility:



$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.8)}{JG}$$

$$T_A = 2.00T_B \qquad [2]$$

Solving Eqs. [1] and [2] yields:

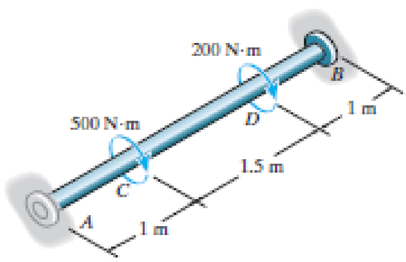
$$T_A = 200 \text{ N}\cdot\text{m} \qquad T_B = 100 \text{ N}\cdot\text{m}$$

Maximum Shear stress:

$$(\tau_{AC})_{max} = \frac{T_A c}{J} = \frac{200(0.025)}{\frac{\pi}{2}(0.025^4)} = 8.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{max} = \frac{T_B c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)} = 4.07 \text{ MPa} \qquad \textbf{Ans.}$$

5–78. The A-36 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



Referring to the FBD of the shaft shown in Fig. a,

$$\Sigma M_x = 0; \quad T_A + T_B - 500 - 200 = 0 \qquad (1)$$

Using the method of superposition, Fig. b

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

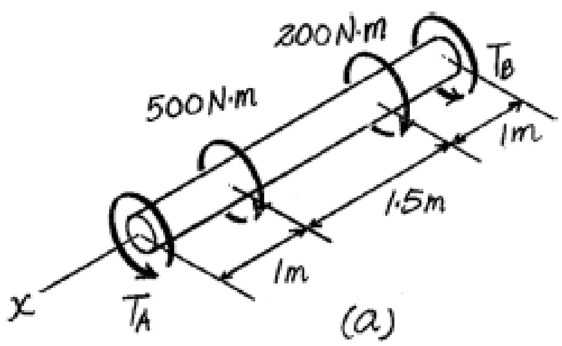
$$T_A = 414.29 \text{ N}\cdot\text{m}$$

Substitute this result into Eq (1),

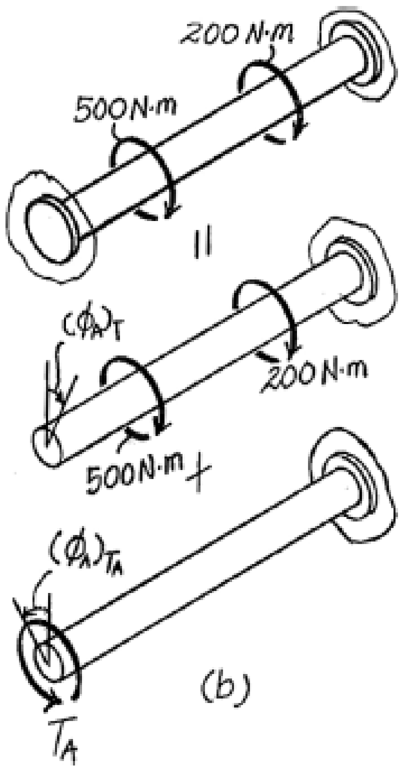
$$T_B = 285.71 \text{ N}\cdot\text{m}$$

Referring to the torque diagram shown in Fig. c, segment AC is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

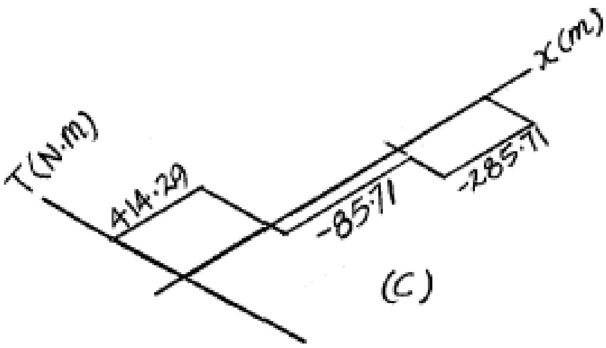
$$\tau_{Abs} = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77 \text{ MPa} \qquad \textbf{Ans.}$$



(a)



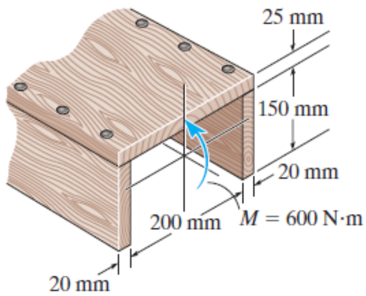
(b)



(c)

CHAPTER 6

6–54. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600 \text{ N} \cdot \text{m}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2\,(0.1)(0.15)(0.2)}{0.24\,(0.025) + 2\,(0.15)(0.02)} = 0.05625 \text{ m}$$

$$I = \frac{1}{12}(0.24)(0.025^3) + (0.24)(0.025)(0.04375^2)$$

$$+ 2\left(\frac{1}{12}\right)(0.02)(0.15^3) + 2(0.15)(0.02)(0.04375^2)$$

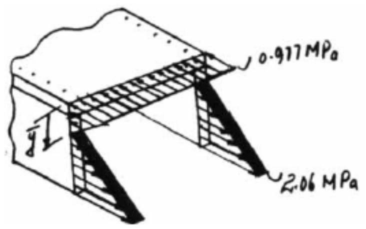
$$= 34.53125\,(10^{-6}) \text{ m}^4$$

$$\sigma_{\max} = \sigma_B = \frac{Mc}{I}$$

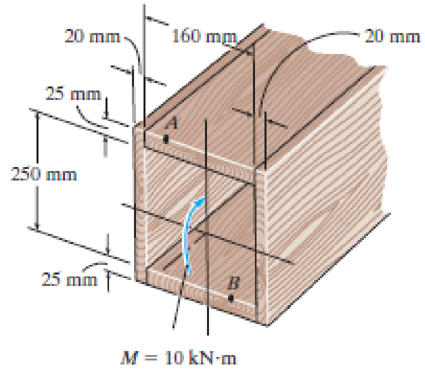
$$= \frac{600\,(0.175 - 0.05625)}{34.53125\,(10^{-6})}$$

$$= 2.06 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{My}{I} = \frac{600\,(0.05625)}{34.53125\,(10^{-6})} = 0.977 \text{ MPa}$$



6–62. A box beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross section is 10 kN · m, determine the stress at points A and B and show the results acting on volume elements located at these points.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.3^3) - \frac{1}{12}(0.16)(0.25^3) = 0.2417(10^{-3})\ \text{m}^4.$$

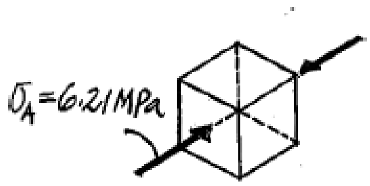
For point A, $y_A = C = 0.15$ m.

$$\sigma_A = \frac{My_A}{I} = \frac{10(10^3)\,(0.15)}{0.2417(10^{-3})} = 6.207(10^6)\text{Pa} = 6.21\ \text{MPa (C)} \qquad \textbf{Ans.}$$

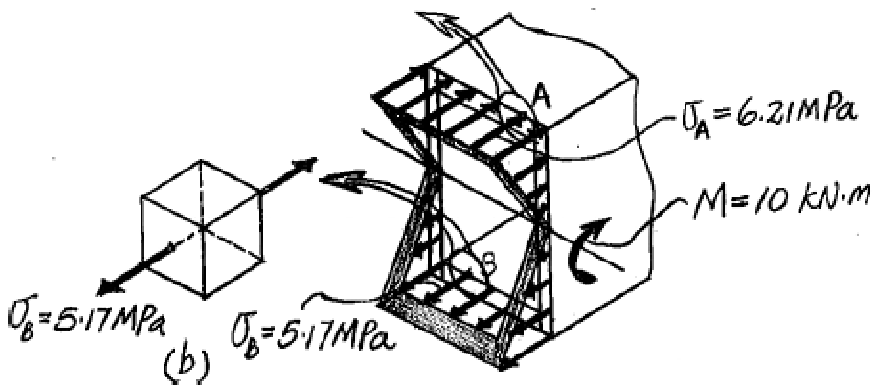
For point B, $y_B = 0.125$ m.

$$\sigma_B = \frac{My_B}{I} = \frac{10(10^3)(0.125)}{0.2417(10^{-3})} = 5.172(10^6)\text{Pa} = 5.17\ \text{MPa (T)} \qquad \textbf{Ans.}$$

The state of stress at point A and B are represented by the volume element shown in Figs. a and b respectively.

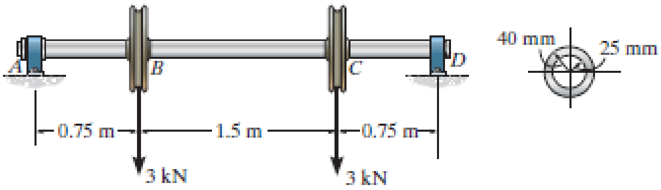


(a)



(b)

6–75. The shaft is supported by a smooth thrust bearing at *A* and smooth journal bearing at *D*. If the shaft has the cross section shown, determine the absolute maximum bending stress in the shaft.



Shear and Moment Diagrams: As shown in Fig. *a*.

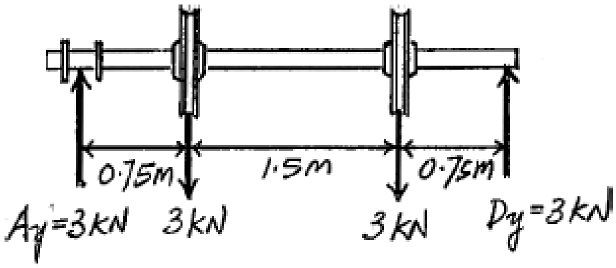
Maximum Moment: Due to symmetry, the maximum moment occurs in region *BC* of the shaft. Referring to the free-body diagram of the segment shown in Fig. *b*.

Section Properties: The moment of inertia of the cross section about the neutral axis is

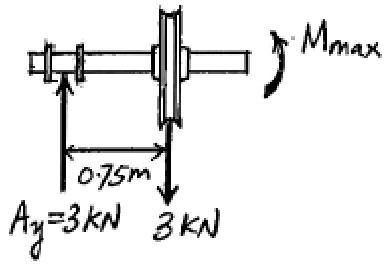
$$I = \frac{\pi}{4}\left(0.04^4 - 0.025^4\right) = 1.7038\left(10^{-6}\right)\text{m}^4$$

Absolute Maximum Bending Stress:

$$\sigma_{\text{allow}} = \frac{M_{\max}c}{I} = \frac{2.25\left(10^3\right)(0.04)}{1.7038\left(10^{-6}\right)} = 52.8 \text{ MPa} \qquad \textbf{Ans.}$$

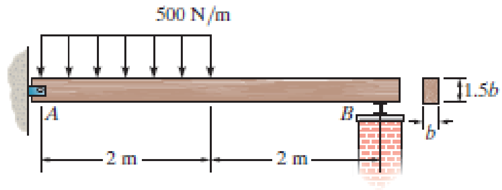


(a)



(b)

6–85. The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension b if the allowable bending stress is $\sigma_{allow} = 10$ MPa.

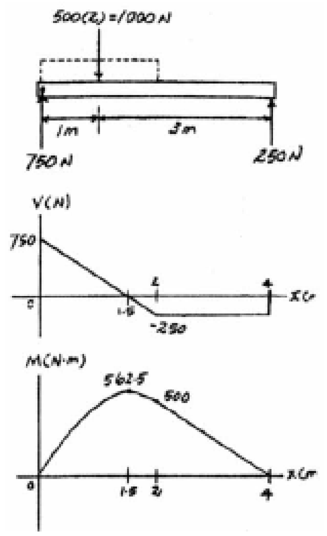


Allowable Bending Stress: The maximum moment is $M_{max} = 562.5$ N · m as indicated on the moment diagram. Applying the flexure formula

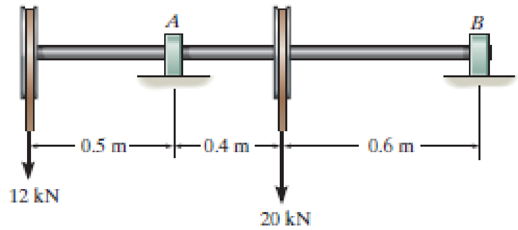
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$10\left(10^6\right) = \frac{562.5(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

$$b = 0.05313 \text{ m} = 53.1 \text{ mm}$$ **Ans.**

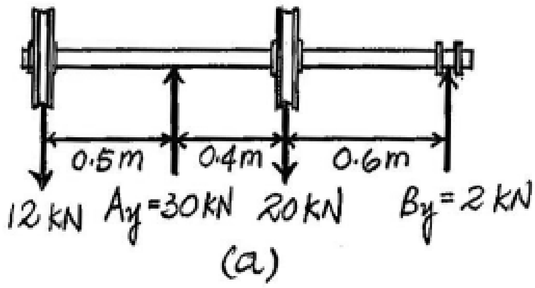


6–91. Determine the absolute maximum bending stress in the 80-mm-diameter shaft which is subjected to the concentrated forces. The journal bearings at *A* and *B* only support vertical forces.



The FBD of the shaft is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{max}| = 6\ \text{kN}\cdot\text{m}$.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\ \text{m}^4$$

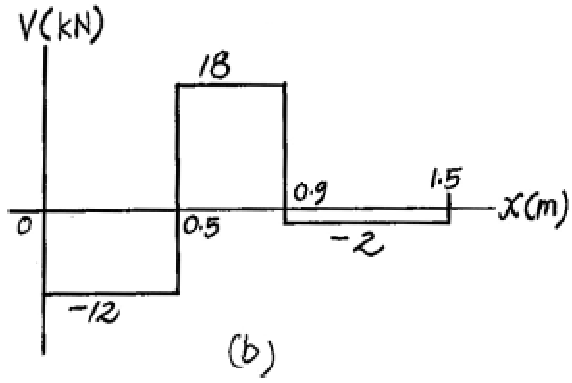
Here, $c = 0.04$ m. Thus

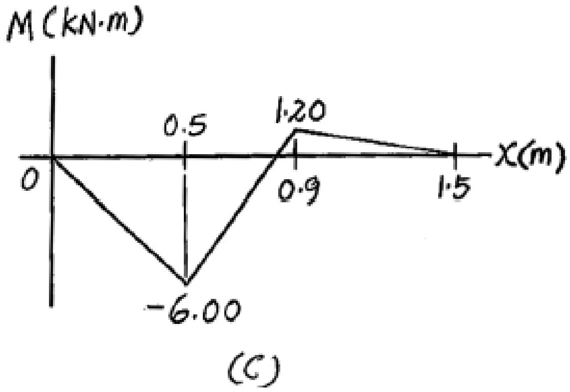
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{6(10^3)(0.04)}{0.64(10^{-6})\pi}$$

$$= 119.37(10^6)\ \text{Pa}$$

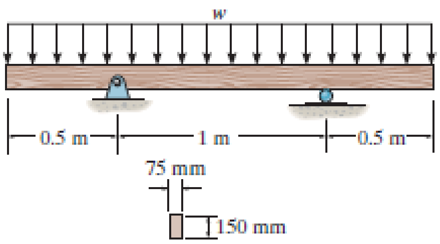
$$= 119\ \text{MPa}$$

Ans.





6–103. Determine the largest uniform distributed load w that can be supported so that the bending stress in the beam does not exceed $\sigma_{allow} = 5$ MPa.



The FBD of the beam is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{max}| = 0.125\,w$.

The moment of inertia of the cross-section is,

$$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6})\text{ m}^4$$

Here, $c = 0.075\,w$. Thus,

$$\sigma_{allow} = \frac{M_{max}\,c}{I};$$

$$5(10^6) = \frac{0.125w(0.075)}{21.09375(10^{-6})}$$

$$w = 11250 \text{ N/m} = 11.25 \text{ kN/m} \qquad \textbf{Ans.}$$